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AUTHOR Porzio, Donald T.
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ABSTRACT

This study sought to gather empirical evidence of the effectiveness of calculus instruction like that used in the Calculus and Mathematica project by examining and comparing the effects of three different instructional approaches to calculus on students' (n=100) abilities to use and understand connections between numerical, graphical, and symbolic representations when solving calculus problems. Data were collected using classroom observations, pre- and posttest instruments, and 36 student interviews. Analysis of the data indicated that: (1) Calculus and Mathematics students were better able to use and to recognize and make connections between different representations than the other students; (2) graphics calculator students were proficient at using graphical representations but had some trouble using symbolic representations and recognizing and making connections between graphical and symbolic representations, even though the use of these representations was stressed during their course; and (3) traditional students were the least proficient at using graphical representations and had the most difficulty recognizing and making connections between different representations. An appendix contains the posttest instrument. Contains 20 references. (MKR)

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Effects of Differing Technological Approaches On Students' Use of Numerical, Graphical and Symbolic Representations and Their Understanding of Calculus

by
Donald T. Porzio
Northern Illinois University

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EFFECTS OF DIFFERING TECHNOLOGICAL APPROACHES ON STUDENTS' USE OF NUMERICAL, GRAPHICAL AND SYMBOLIC REPRESENTATIONS AND THEIR UNDERSTANDING OF CALCULUS

Donald T. Porzio
Northern Illinois University
dporzio@math.niu.edu

There is a growing body of research on the effects of using different forms of technology, such as graphics calculators, computer graphing utilities, and computer algebra systems, in calculus instruction (e.g., Beckmann, 1988; Crocker, 1991; Emese, 1993; Heid, 1985; Palmiter, 1986; Stout, 1991). Most of this research has examined how using technology to emphasize concept understanding while de-emphasizing routine computational skills impacts upon students' conceptual and procedural knowledge (e.g., Hawker, 1987; Heid, 1985; Judson, 1989, Palmiter, 1986; Schrock, 1990). Little research has been done on the impact of technology use in calculus instruction on other aspects of students' understanding of calculus, such as their understanding of different representations of calculus concepts or their ability to use different representations when solving problems (Beckmann, 1988; Hart, 1991). In addition, there is a lack of research on the impact of the instructional methods in calculus reform projects such as Calculus & *Mathematica* that emphasize the use of technology, multiple representations, and the solving or interpreting of specific problems as a means of developing conceptual and procedural knowledge (Crocker, 1991; Park, 1993). The present study sought to gather empirical evidence of the effectiveness of calculus instruction like that used in the Calculus & *Mathematica* project by examining and comparing the effects of three different instructional approaches to calculus on students' abilities to use and understand connections between numerical, graphical, and symbolic representations when solving calculus problems. The following three research questions were investigated in this study:

1. What is the relationship between the instructional approach that students experience and any change in their initial preference for different representation when solving calculus problems?
2. What is the relationship between the instructional approach that students experience and their abilities to use graphical, numerical, and symbolic representations when solving calculus problems?
3. What is the relationship between the instructional approach that students experience and their abilities to see, or make, connections between graphical, numerical, and symbolic representations in the context of problem situations?

METHODOLOGY

Participants for this study were undergraduate students chosen from intact classes from three different calculus courses at a large midwestern university during Autumn Quarter 1993. Each course was the first in a four-quarter calculus sequence designed primarily for mathematics, science, and engineering students. One course, Math 151, used a traditional approach to calculus instruction that emphasized use of symbolic representations. The second course, Math 151G, was similar in content to Math 151 but the instruction and assignments for this course stressed use of symbolic representations and graphical representations generated via graphics calculators. The third course, Math 151C, used the electronic calculus course Calculus & *Mathematica* (Davis, Porta, and Uhl, 1994). In this course, instruction and assignments emphasized use of symbolic, numerical, and graphical representations and the solving of problems designed to establish or reinforce connections between different representations or between concepts and procedures.

Data were collected using classroom observations, pre- and posttest instruments, and 36 student interviews. Weekly classroom were made by the researcher to document the use of multiple representations in each course by both the instructors and the students. The pretest and posttest measured students' initial preferences for certain representations when solving problems. The interviews and posttest were used to evaluate students' use and understanding of different representations when solving calculus problems. In order to make the comparison between students from such distinctly different types of calculus courses as equitable as possible, the problems on the posttest were designed by the researcher so that they would be solvable for any calculus students, no matter which course they completed. These problems (see Appendix A) were pilot tested twice prior to the beginning of the study and content validity was established by a panel of mathematicians and mathematics educators from across the United States.

The posttest instrument was given during the final week of classes to 100 students from Math 151 (n = 40), Math 151G (n = 24) and Math 151C (n = 36). Because of time limitations, each student was asked to solve only two of the four problems on the posttest using whatever means they normally use to solve homework problems. All students were assigned problem 1. The other problem was assigned randomly so that approximately the same number of students from each course attempted problems 2, 3, and 4. From the group of 100 students who took the posttest, 12 volunteers from each course were chosen to participate in one-on-one interview with the researcher. The interviews were used to gather additional information on students' use and understanding of different representations that might not have been discerned by simply looking at their solutions to the posttest problems. The interviews took place during Spring Quarter 1994 and lasted

between 25 to 45 minutes. Students were paid for participating in the interviews since the interviews were conducted during students' free time.

THEORETICAL FRAMEWORK

A theoretical framework for analyzing differences in students' abilities to use and understand connections between different representations was developed from theories put forth by Hiebert and Carpenter (1992) and Dubinsky (1991). Hiebert and Carpenter propose a theoretical framework for understanding based on the premise that knowledge is represented internally, or mentally, in some structured form. They suggest that when connections between internal representations are constructed, these representations and connections form networks of knowledge. Furthermore, they posit that a mathematical concept is understood if its internal representation is part of an internal network of knowledge and that the degree of understanding of the concept is determined by the number and strength of the connections in the internal network. Under this framework, differences in students' abilities to use and understand connections between numerical, graphical, and symbolic representations thought to be related to the different instructional approaches of the calculus courses can be analyzed in terms of differences in internal networks of represented knowledge likely to be formed by students in the different courses.

Dubinsky (1991) applies Piaget's notion of reflective abstraction to advanced mathematical thinking to form a theory of mathematical knowledge and its construction or acquisition. According to Dubinsky, reflective abstraction, in part, consist of the construction of schema, which are comprised of mental objects and mental actions on these objects, and it occurs when students are constructing new knowledge by solving and interpreting problems. When problem solving is successful, the student assimilates the problem and its solution into one or more schema. When the problem solving is not successful then the student may or may not make accommodations in existing schema to handle the unsolved problem situation. The theoretical framework based on Dubinsky's (1991) theory was used to explain differences in students' abilities to use and understand connections between numerical, graphical, and symbolic representations thought to be related to the different instructional approaches of the calculus courses that could not be adequately explained in terms of differences in internal networks of represented knowledge likely to be formed by students in the different courses. In particular, Dubinsky's theory helped to explain differences in use and understanding of different representations amongst students who experienced instructional approaches that emphasized the use of different representation when presenting concepts or solving problems.

RESULTS

Analysis of the data from this research indicate that (a) Calculus & *Mathematica* students were better able to use and to recognize and make connections between different representations than the other students, (b) graphics calculator students were proficient at using graphical representations but had some trouble using symbolic representations and recognizing and making connections between graphical and symbolic representations, even though use of these representations were stressed during their course, and (c) traditional students were the least proficient at using graphical representations and had the most difficulty recognizing and making connections between different representations. These findings suggest that an instructional approach that emphasizes the use of multiple representations of concepts and that includes opportunities for students to solve problems specifically designed to explore or establish connections between representations has the greatest impact on students' abilities to use or to recognize connection between different representations. These results are not surprising when analyzed in terms of the study's theoretical framework.

The instruction approach experienced by the traditional students emphasized the use of symbolic representations to present concepts and solve problems. Little use was made of graphical or numerical representations. As such, it was reasonable to expect that traditional students would form disjoint or weakly-connected internal networks of knowledge that would make it difficult for them to use anything but symbolic representations or to recognize and make connections between anything but symbolic representations.

The instruction approach experienced by the Calculus & *Mathematica* students emphasized the use of numerical, graphical, and symbolic representations to present concepts and solve problems. They were also required to interpret and solve various related problems specifically designed to examine, explore, or establish connections between different representations of the same concept. As such, it was reasonable to expect that Calculus & *Mathematica* students would form well-connected internal networks of knowledge that would allow them to make use of different types of representations and to recognize and make connections between various different representations.

The instruction approach experienced by the graphics calculator students emphasized the use of graphical and symbolic representations to present concepts and solve problems. Students were expected to use graphical representations to confirm results found analytical and to use symbolic representations to confirm results found graphically, but were given few problems specifically designed to have them make connections between the graphical and symbolic representations they used when solving problems. As such, it was reasonable to expect that graphics calculator students *might* form well-connected internal networks of knowledge that would allow them to make use of different types of representations and to recognize and make connections between various different representations. However, since the graphics calculators students had little opportunity for

the type of reflective abstraction Dubinsky (1991) suggests is necessary if students are to construct, or acquire, new knowledge, it was also reasonable to expect that some graphics calculator students would form weakly-connected internal networks of knowledge that would make it difficult for them to recognize and make connections between graphical and symbolic representations. In addition, interviews with the graphics calculator students indicate that they perceived the focus of the course to be on the use of graphical representations, not on the use of both graphical and symbolic representations. This perception may have influenced the development of students' internal networks of knowledge related to symbolic representations and may provide an alternate explanation as to why the graphics calculator students had difficulties using symbolic representations and recognizing and making connections between graphical and symbolic representations.

CONCLUSIONS

Several recommendations arise from findings of this research in the spirit of the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), the *Professional Standards for Teaching Mathematics* (NCTM, 1991), and *A Call for Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics* (Leitzel, 1991). First, the NCTM *Standards* (1989) calls for a mathematics curriculum that "emphasizes conceptual understanding, multiple representations and connections, mathematical modeling, and mathematical problem solving" (p. 125). Based on this study, it seems apparent that parts of the curriculum must be taught in concert, not in isolation. Emphasis on multiple representations and connections alone appears to help but *not* necessarily promote understanding of connections between representations. Emphasis on multiple representations and connections established in a problem-solving setting designed to help students explore these representations and connections does appear to promote understanding of connections between representations.

The first recommendation forms the basis for another recommendation. The two *Standards* (NCTM, 1989, 1991) and *A Call for Change* (Leitzel, 1991) all recognize the importance of the teacher in accomplishing the type of reform to mathematics curriculum suggested in the former document. Therefore, teachers need to be prepared to help students make connections between different representations of the same concept, not by simply showing them the connections, but working problems with them that explore, establish, and reinforce the connections. Teachers also need to avoid the novice teacher "trap" of not making connections themselves (Leinhardt, 1984) by being aware of, and understanding connections between representations so they will not miss opportunities to make these connections explicit for students.

The results from this study indicate that the addition of a technological component to the existing calculus curriculum to provide easier access to representations may not necessarily improve students' understanding of calculus. This suggests that revisions to calculus

curriculum, including revisions to calculus textbooks, should be done in such a way that multiple representations, connections, *and* technology are *not* simply tacked onto the existing topics and problems, but are woven into a set of new topics and problems that emphasize using of multiple representations, recognizing connections between representations, and making appropriate use of available technology.

Finally, recent technological advancements have lead to the emergence of software that allows for multiple, dynamically linked representations (see Kaput, 1992, 1993, for example). This study suggests that research is needed to determine how students develop connections between representations that are "linked" by the software. Will students view these connections as if they were being presented by an instructor, or will they explore and establish the connections themselves as they might if the connections were not made explicit by the software? Research will be needed to determine what type of instruction and problems are needed to make the best use of the dynamically linked representations. Instruction and problems designed to help students explore and establish connections between different representations will have to change in light of software that will establish the connections for students.

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APPENDIX A - POSTTEST INSTRUMENT

1. The population of a herd of deer is given by the function

$$P(t) = 4000 - 500(\cos 2\pi t)$$

where t is measured in years and $t = 0$ corresponds to January 1.

   a. When in the year is the population at its maximum? What is that maximum?

   b. When in the year is the population at its minimum? What is that minimum?

   c. When in the year is the population increasing the fastest?
   When in the year is the population decreasing the fastest?

   d. Approximately how fast the population is changing on the first of July?

2. Suppose N, the <u>total</u> number of people who have contracted a disease t days after its outbreak, is given by the formula $N = \dfrac{1{,}000{,}000}{1 + 5000e^{-0.1t}}$.

   a. In the long run, how many people will contract the disease?

   b. Is there a maximum number of people who will eventually contract the disease? Explain.

   c. Is there any day on which more than a million people fall sick? Half a million? Quarter of a million? (Note: You do not need to determine on what days these things happen.)

3. The table below gives U.S. population figures between 1790 and 1980.

Year	Population (in millions)	Year	Population (in millions)	Year	Population (in millions)	Year	Population (in millions)
1790	3.9	1840	17.1	1890	62.9	1940	131.7
1800	5.3	1850	23.1	1900	76.0	1950	150.7
1810	7.2	1860	31.4	1910	92.0	1960	179.0
1820	9.6	1870	38.6	1920	105.7	1970	205.0
1830	12.9	1880	50.2	1930	122.8	1980	226.5

   a. Approximately how fast was the population changing in the years 1900, 1945, and 1980?

   b. During what year(s) does it appear that the population growth was the greatest? Explain.

   c. Based on this data, what population would you predict for the 1990 census?

4. a. Show that $x > 2 \ln x$ for all $x > 0$.
   (Note: This is equivalent to showing that $e^x > x^2$ for all $x > 0$.)

   b. Is it true that $x > 3 \ln x$ for all $x > 0$?
   If not, estimate M such that $x > 3 \ln x$ for all $x > M$.

   c. What would you predict is the largest value of a for which $x > a \ln x$ for all $x > 0$?
   (Note: This is equivalent to predicting the largest value of a for which $e^x > x^a$ for all $x > 0$.)